Gisselquist
Technology, LLC

DOUBLE CLOCKED FFT SPECIFICATION

Dan Gisselquist, Ph.D.
dgisselq@opencores.org

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Revision History

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0.1	3/3/2015	Gisselquist	First Draft

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Preface

This FFT comes from my attempts to design and implement a signal processing algorithm inside a generic FPGA, but only on a limited budget. As such, I don't yet have the FPGA board I wish to place this algorithm onto, neither do I have any expensive modeling or simulation capabilities. I'm using Verilator for my modeling and simulation needs. This makes using a vendor supplied IP core, such as an FFT, difficult if not impossible to use.

My problem was made worse when I learned that the published maximum clock speed for a device wasn't necessarily the maximum clock speed that I could achieve. My design needed to process the incoming signal at 500 MHz to be commercially viable. 500 MHz is not necessarily a clock speed that can be easily achieved. 250 MHz, on the other hand, is much more within the realm of possibility. Achieving a 500 MHz performance with a 250 MHz clock, however, requires an FFT that accepts two samples per clock.

This, then, was and is the genesis of this project.

Dan Gisselquist, Ph.D.

1.

Introduction

The Double Clocked FFT project contains all of the software necessary to create the IP to generate an arbitrary sized FFT that will clock two samples in at each clock cycle, and after some pipeline delay it will clock two samples out at every clock cycle.

The FFT generated by this approach is very configurable. By simple adjustment of a command line parameter, the FFT may be made to be a forward FFT or an inverse FFT. The number of bits processed, kept, and maintained by this FFT are also configurable. Even the number of bits used for the twiddle factors, or whether or not to bit reverse the outputs, are all configurable parts to this FFT core.

These features make the Double Clocked FFT very different and unique among the other cores available on opencores.com.

For those who wish to get started right away, please download the package, change into the `sw` directory and run `make`. There is no need to run a configure script, `fftgen` is completely portable C++. Then, once built, go ahead and run `fftgen` without any arguments. This will cause `fftgen` to print a usage statement to the screen. Review the usage statement, and run `fftgen` a second time with the arguments you need.

2.

Generation

Creating a double clocked FFT core is as simple as running the program `fftgen`. The program will then create a series of Verilog files, as well as `.hex` files suitable for use with a $readmemh, and place them into an `./fft-core/` directory that `fftgen` will create. Creating the core you want takes a touch of configuring. Therefore, the following lists the arguments that can be given to `fftgen` to adjust the core that it builds:

-f size This specifies the size of the FFT core that `fftgen` will build. The size must be a power of two. The transform is given, within a scale factor, to,

$$X[k] \quad = \quad \sum_{n=0}^{N-1} x[n] \, e^{-j2\pi \frac{k}{N} n}$$

-1 This specifies that the FFT will be an inverse FFT. Specifically, it will calculate,

$$x[n] \quad = \quad \sum_{k=0}^{N-1} X[k] \, e^{j2\pi \frac{k}{N} n}$$

-0 This specifies building a forward FFT. However, since this is the default, this option never necessary.

-s This causes the core to skip the final bit reversal stage. The outputs of the FFT will then come out in bit reversed order.

   This option is useful in those cases where someone wishes to multiply the coefficients coming out of an FFT by some product, and then to inverse FFT the results. If the coefficients are also applied in bit–reversed order, then both the FFT and IFFT may skip their bit reversals.

-S Include the final bit reversal stage. As this is also the default, specifying the option should not be necessary.

-d DIR Specifies the DIRectory to place the produced Verilog files. By default, this will be in the './fft-core/' directory, but it can be moved to any other directory as necessary.

-n bits Sets the number of input bits per sample. Given this setting, each of the two samples clocked in at every clock cycle will have this many bits for their real portion, and again this many bits for their imaginary portion. Thus, the data input to the FFT will be four times this many bits per clock.

-m bits This sets the maximum bit width of the output. By default, the FFT will gain bits as they accumulate within the FFT. Bits are accumulated at roughly one bit for every two stages. However, if this value is set, bits are only accumulated up to this maximum width. After this width, further accumulations are truncated.

-c bits The number of bits in each twiddle coefficient is given by the number of bits input to that stage plus this extra number of bits per coefficient. By increasing the number of bits per coefficient above that of the input samples, truncation error is kept to the original error found within the original samples.

3.

Architecture

As a component of another system the structure of this system is a simple black box such as the one shown in Fig. 3.1. The interface is simple: strobe the reset line, and every clock thereafter set the clock enable line when data is valid on the left and right input ports. Likewise for the outputs, when the **o_sync** line goes high the first data sample is available. Ever after that, one data sample will be available every clock cycle that the **i_ce** line is high.

Internal to the FFT, things are a touch more complex. Fig. 3.2 attempts to show some of this structure. As you can see from the figure, the FFT itself is composed of a series of stages. These stages are split from the beginning into an even stage and an odd stage. Further, they are numbered according to the size of the FFT they represent. Therefore the first stage is numbered N and represents the first stage of an N point FFT. The second stage is labeled $N/2$, then $N/$, and so on down to $N = 8$. The four sample stage and the two sample stages are different, however. These two stages, representing three blocks on Fig. 3.2, can be accomplished without any multiplies. Therefore they have been accomplished separately. Likewise all of the stages, save the double stage at the bottom, operate on one data sample per clock. Only the last stage, prior to the bit reversal stage, takes two data samples per clock as input, and outputs two data samples per clock. Finally, the bit reversal stage acts as the last piece of the structure.

Internal to each of the FFT stages is a butterfly and a complex multiply, as shown in Fig. 3.3. These FFT stages are really no different than any other decimation in frequency FFT, save only that the coefficients are alternated between the two stages. That is, the even stages get all the even coefficients, and the odd stages get all of the odd coefficients. Internally, each stage spends the first $N/4$ clocks storing its inputs into memory, and then the next $N/4$ clocks pairing a stored input with a single external input, so that both values become inputs to the butterfly. Likewise, the butterfly coefficient is read from a small ROM table.

One trick to making the FFT stage work successfully is synchronization. Since the multiplies create a delay of (roughly) one clock cycle per bit of input, there is a significant pipeline delay from the input to the output of the butterfly routine. To match this delay, the FFT stage places a synchronization pulse into the butterfly. When this synchronization pulse comes out of the butterfly, the values of the butterfly then match the first sample out of the stage. The next synchronization problem comes from the fact that the butterflies operate on two samples at a time, whereas the FFT stage operates on a single sample at a time. This means that half the time the butterfly output will be invalid. To keep things aligned, and to avoid the invalid data half, a counter is started by the synchronization pulse coming out of the butterfly in order to keep track. Using this counter and once the butterfly produces the first sync pulse, the next $N/4$ clock cycles will produce valid
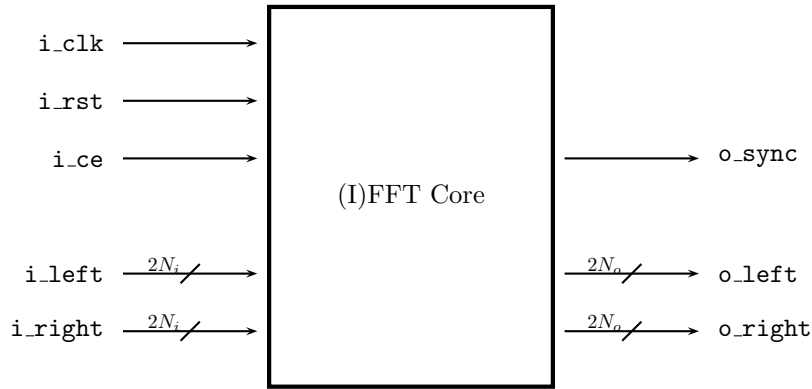
Figure 3.1: (I)FFT Black Box Diagram

butterfly outputs. For these clock cycles, the left or first output is sent immediately to the next FFT stage, whereas the right or second output is saved into memory. Once these cycles are complete, the butterfly outputs will be invalid for the next $N/4$ clock cycles. During these invalid clock cycles, the FFT stage outputs data that had been stored in memory. In this fashion, data is always valid coming out of each FFT stage once the initial synchronization pulse goes high.

The complex multiply itself, formed internal to the butterfly routine, is formed from three very simple shift and add multiplies, whose output is then transformed into a single complex output. To avoid overflow, the complex coefficients, z_n, for these multiplies are given by,

$$z_n = c_n + js_n, \text{ where} \tag{3.1}$$

$$c_n = \left\lfloor 2^{C-2}\cos\left(2\pi\frac{n}{N}\right) + \frac{1}{2}\right\rfloor, \tag{3.2}$$

$$s_n = \left\lfloor 2^{C-2}\sin\left(2\pi\frac{n}{N}\right) + \frac{1}{2}\right\rfloor, \text{ and} \tag{3.3}$$

C is the number of bits allocated to the coefficient.

For those wishing to understand this operation further and in more depth, I would commend them to the literature on how a decimation in frequency FFT is constructed.
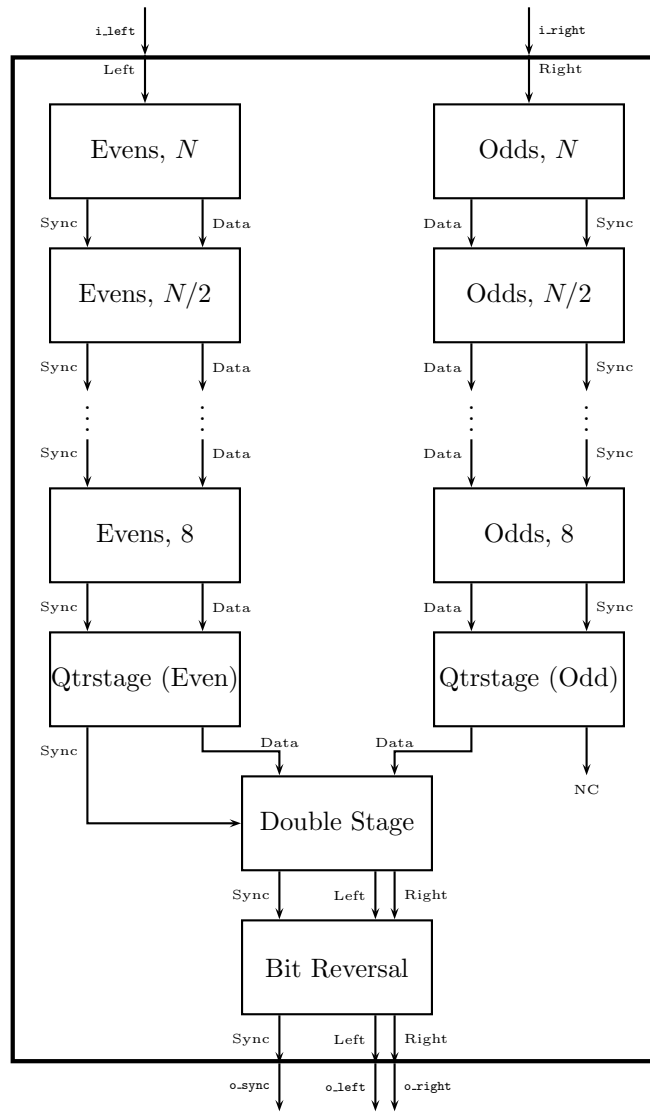
i_left   i_right

Left   Right

| Evens, N | | Odds, N |

Sync   Data   Data   Sync

| Evens, $N/2$ | | Odds, $N/2$ |

Sync   Data   Data   Sync

⋮   ⋮   ⋮   ⋮

Sync   Data   Data   Sync

| Evens, 8 | | Odds, 8 |

Sync   Data   Data   Sync

| Qtrstage (Even) | | Qtrstage (Odd) |

Sync   Data   Data

NC

| Double Stage |

Sync   Left   Right

| Bit Reversal |

Sync   Left   Right

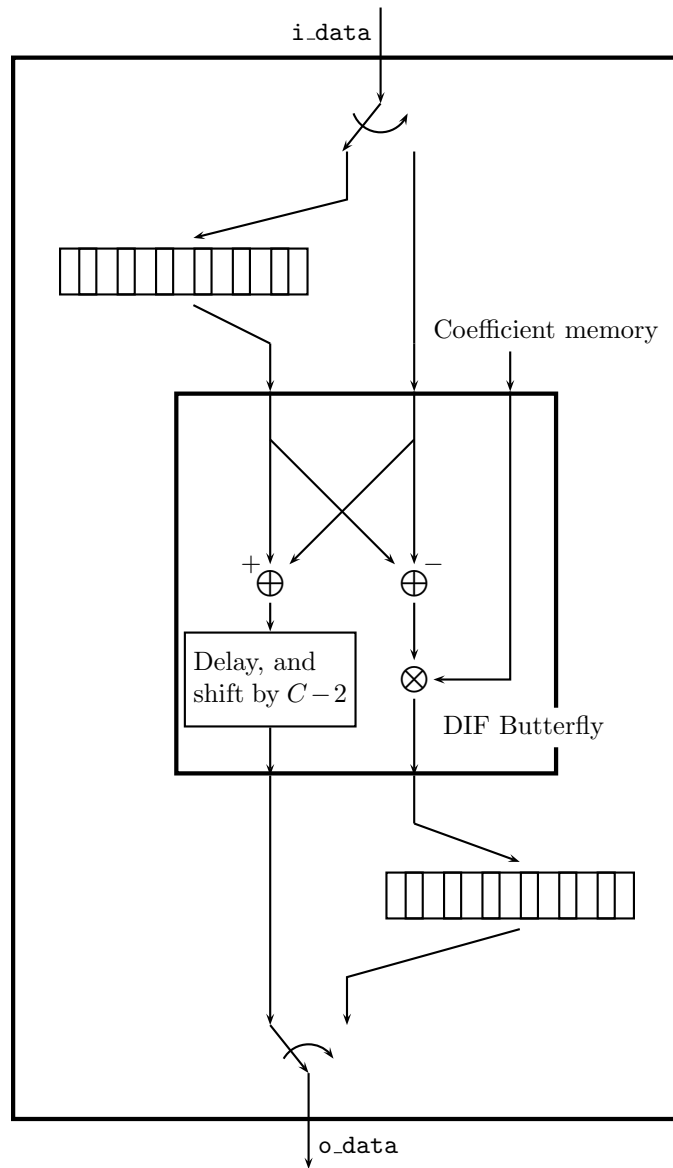o_sync   o_left   o_right

Figure 3.2: Internal FFT Structure

Figure 3.3: A Single FFT Stage, with Butterfly

4.

Operation

The core is actually really easy to use:

1. Provide a system clock to the core every clock cycle.

2. Set the `i_rst` line high for at least one clock cycle before you intend to use the core.

3. From the time of reset until the first sample pair is available on the IO ports, `i_rst` may be kept low, but the clock enable line `i_ce` must also be kept low.

4. On the clock containing the first sample pair, `i_left` and `i_right`, set `i_ce` high.

5. Ever after, any time a valid pair of samples is available to the input of the FFT, place the first sample of the pair on the `i_left` line, the second on the `i_right` line, and set `i_ce` high.

6. At the first valid output, the FFT core will set `o_sync` line high in addition to the output values `o_left` (the first of two), and `o_right` (the second of the two).

7. Ever after, whenever `i_ce` is high, the FFT core will clock two samples in and two samples out. On any valid first pair of samples coming out of the transform, `o_sync` will be high. Otherwise `o_sync` will remain low.

There are no special modes or states associated with this core. If you wish it to stop or pause, just turn off `i_ce`. If you wish to flush the core, just send zeros into the core.

5.

Registers

Once built, the FFT routine has no capability for runtime configuration or reconfiguration. Therefore, this implementation maintains no user configurable or readable registers.

This is a great advantage in many ways, simply because it greatly simplifies the interface over other cores that are available out there.

6.

Clocks

The FFT routines built by this core use one clock only. The speed of this clock will depend upon the speed your hardware is capable of. If your data rate is slower than your clock speed, just hold off on the `i_ce` line as necessary so that every clock with the `i_ce` line high is a valid sample.

7.

IO Ports

The FFT core presents a small set of IO ports to its external interface. These ports are listed in Table. 7.1.

Port	Width	Direction	Description
i_clk	1	Input	The global clock driving the FFT.
i_rst	1	Input	An active high synchronous reset.
i_ce	1	Input	Clock Enable. Set this high to clock data in and out.
i_left	$2N_i$	Input	The first of two input complex input samples. Bits $[2N_i - 1:2N_i]$ of this value are the real portion, whereas bits $[2N_i - 1:0]$ represent the imaginary portion. Both portions are in signed twos complement integer format. The number of bits, N_i, is configurable.
i_right	$2N_i$	Input	The second of two input complex input samples. The format is the same as `i_left` above.
o_left	$2N_o$	Output	The first of two input complex output samples. The format is the same, save only that N_o bits are used for each twos complement portion instead of N_i.
o_right	$2N_o$	Output	The second of two input complex output samples. The format is the same as for `o_left` above.
o_sync	1	Output	Signals the first output sample pair of any transform, zero otherwise.

Table 7.1: List of IO ports